Mathematical Modelling and Non-linear Analysis of Thermal Convection with Rotational Modulation in Viscoelastic Ferromagnetic Fluids

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Abstract

The non-linear instability of viscoelastic ferromagnetic fluid with the modulated rotational speed heated from below is studied. Fourier series with a minimal representation has been used for the study. A time-periodic and sinusoidally varying rotational modulation was considered. Using numerical technique of Runge–Kutta–Fehlberg45, the solution of the resulting Khayat–Lorenz model was obtained to quantify heat transport. Rotational modulation effect is examined for different parameter variation. The results of three types of fluids namely Newtonian, Rivlin-Ericksen and Maxwell are obtained as particular cases of present study. The modulated centrifugal force has many industrial applications like solidification of alloys and rotating turbo machinery.

Keywords*: Rotational modulation, Khayat-Lorenz model, Viscoelastic Ferromagnetic, Heat Transport, Mean Nusselt Number*

1.0 Introduction

Ferromagnetic fluids have importance in industry and modern technology. These fluids provided basis for electro mechanical and chemical devices, for instance, rotating X-ray tubes, electromagnets, generators, transformers, electric engines and recording procedures, while in biological sciences, one can use in magnet therapy for gout, migraines and headaches, pain management, cure arthritis, magnetic resonance imaging etc. Stability of these fluids was studied by Rosensweig [1]. It has been well established that Rayleigh-Bénard convection (RBC) occurs due to bottom heating [2].

Ferrofluid convection has been addressed by many authors [3-9]. Viscoelasticity is the behavior of materials with both fluid and elastic characteristics simultaneously. This property is a result of temporary connections between the fiber-like particles. A polymer always exhibits

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viscoelastic behavior since it is composed of long molecules that are capable of temporary connections with nearby molecules. Most of the biofluids are viscoelastic. Measurement of viscoelasticity is therefore useful for clinical investigation and artificially controlling mucous secretion via viscoelasticity is the goal of drug research. It becomes clear that such studies have a significant social impact. Viscoelastic fluids are considered as working media for many practical problems [10]. Also, many works on thermal convection in viscoelastic fluid convection are available in literature [11-13].

However, a few applications and studies of magnetic viscoelastic fluids (MVF) or Viscoelastic ferromagnetic fluids (VFF) are available in open literature. Laroze et al. [14] and Pérez et al.[15, 16] have reported theoretical study of convection via oscillatory and stationary modes in VFF.

Traditional heat transfer fluids such as water, oil and ethylene glycol cause problems in the performance of engineering equipment such as heat exchangers and electronic devices due to their low thermal conductivity. Study of heat transfer and convection in different types of fluids are important. Many articles are found for regulating thermal convection onset and also heat transport in the non-isothermal application of viscoelastic/ferromagnetic fluids [17-21].

Almost all fluid mechanics engineering applications are applied to systems in which external forces play a significant role. In some cases, they may be characterized as rotational speed, gravity and temperature modulations. Many researchers studied the RBC under gravity/temperature modulation [22-26].It is widely known that external rotation can affect fluid flow by significantly altering the nature of it through the Coriolis Effect. In fluids, convectional heat transfer and rotational processes play an important role. This field of study has a number of applications in modern science, such as turbo machines, enlarged oil production, rotating atomic scraps repository, geothermal energy utilization, insulation engineering etc. Bhadauria and Kiran [27] made an analysis of the nonlinear stability of a rotating temperature-dependent viscous fluid layer with a rotating internal heating and rotation speed modulation.

In Oldroyd-B fluids with double diffusive convection, the effect of rotation modulation is analyzed by Vanishree and Anjana [28]. Both linear and non-linear stability was analysed. The thermal Rayleigh number was computed using a regular perturbation technique. The results indicate that the strain retardation parameter and Lewis number stabilize the system while stress relaxation destabilizes it. Kanchana et al. [29] investigated the effect of in-phase and out-of-phase temperature modulations, time-periodic gravity modulation and rotational modulation on Rayleigh-Bénard convection in 28 nano liquids. By examining RBC with a rotational modulation, Anjana et al. [30] studied the linear and nonlinear behavior of Oldroyd fluids. Based on the cited literature, it appears that most research on rotation modulation has been done on viscoelastic or ferromagnetic fluids. There is no nonlinear study available in the literature that considers the effect of rotation speed modulation in VFF, to the best of the authors' knowledge. The present research was carried out to analyze weakly nonlinear stability in a VFF with the effects of rotational speed modulation.

2.0 Mathematical Formulation

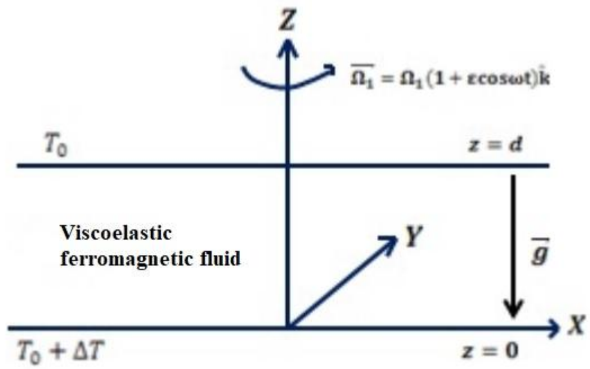


Fig. 1. Physical configuration

This model considers an infinitely thin layer of VFF with a thickness d and a gravitational field$\boldsymbol{g}$. Constant temperatures are maintained in the upper plate and lower plate as $T_o + \Delta T$ at $z = d$and $T_o\ at\ z = 0$ respectively. Without cooling or heating ambient temperature is T_o. It is assumed thatthe fluid layer rotates about the z-axis with a variable rotational speed$\overrightarrow{\Omega_1} = \Omega_1(1 + \varepsilon cos\omega t)\hat{k}$. In this case, only heating from below is taken into account. Fig.1 is a schematic representing the flow configuration. Based on the Boussinesq approximation, the following are the equations governing the phenomena.

An incompressible fluid's continuity equation is:

$$q_{i,i} = 0 \qquad (1)$$

The momentum equation for an incompressible fluid is:

$$\rho_0\left(\frac{\partial q_{\ i}}{\partial t}+q_j\, q_{\ i,j}\right)=-p,i\ +\ \left[\frac{\rho_0\left|\overrightarrow{\Omega_1}\times\vec{r}\right|^2}{2}\ -p\right]_{,\ i}+\tau'_{ij\ ,j}\ +\ \rho g_i+$$

$$+2\rho_0\varepsilon_{ijk}q_j\,\Omega_{1k}+\mu_0\left(M_jH_{i,j}\right)\qquad(2)$$

Constitutive Equation is:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\tau'_{ij}\ =\ \left[q_{\ i,j}+q_{\ j,i}+\lambda_2\frac{\partial}{\partial t}\left(q_{\ i,j}+q_{\ j,i}\right)\right][\mu].$$

The temperature equation for an incompressible fluid is:

$$\frac{\partial T}{\partial t}+\ q_jT_{,j}\ -\ \kappa\left[T_{,j}\right]_{,j}=0.\qquad(3)$$

For a Boussinesq magnetic fluid, the density equation of state is:

$$\rho=-\rho 0[\alpha(T-T0)-1].\qquad(4)$$

In the case of non-conducting fluids, Maxwell’s equations are as follows:

$$B_{i\ ,i}=0\qquad\text{and}\ \ \epsilon_{ijk}H_{k,j}=0.\qquad(5)$$

Further the magnetic field H_i, magnetization M_i and magnetic induction B_i are related by

$$B_i=\ \mu_0(M_i+\ H_i).\qquad(6)$$

The magnetization depends on the magnitude of the magnetic field and temperature which can be expressed as

$$M_i=\ \frac{M_0}{H_0}H_i.\qquad(7)$$

In order to evaluate the partial derivatives of magnetization M, the linearized magnetic equation of state (Finlayson[31]) for a single component fluid is

$$M=[T-T_0]k_l+M_0+[H-H_0]\chi_m,\qquad(8)$$

where the physical variables are indicated in table 1.

Table 1. Nomenclature

T_a Taylor number	**Greek symbols:**
P_r Prandtl number	Ω_1 angular velocity
H_0 applied magnetic field	ρ_0 reference density (kg/m^3)
d thickness of fluid layer(m)	ε amplitude of modulation
g_i gravitational acceleration (0, 0, -g) (m/s^2)	α thermal expansion coefficient (K^{-1})
k non dimensional wave number (m^{-1})	κ thermal diffusivity (m^2/s)
B_i magnetic induction	δ small positive constants
p effective pressure	μ viscosity $(kgm^{-1}s^{-1})$
Nu Nusselt number	ρ density (kg/m^3)
M_1 buoyancy magnetic number	ω frequency (s^{-1})
M_3 non-buoyancy magnetic number	λ_2 coefficient of strain retardation(s)
R Rayleigh number	λ_1 coefficient of stress relaxation(s)
t time(s)	Λ ratio of elasticity
T temperature(K)	Λ_1 scaled stress relaxation parameter (Deborah number)
T_0 constant temperature of the boundary (K)	Λ_2 strain retardation parameter (scaled)
k_l pyromagnetic coefficient	τ_{ij} stress components (N/m^2))
q_i components of velocity (m/s) (u,v,w)	χ_m magnetic susceptibility
Subscripts:	ϕ magnetic scalar potential
b basic state	**Superscripts:**
0 reference value	′ perturbedquantity
	* dimensionless quantity

3.0 Basic State

A basic state consists of the following quantities:

$$M_b(z)= \frac{k_l}{1+\chi_m}\left(\frac{z}{d}-1\right)+\mathrm{M}_0,\ H_b(z)= \frac{k_l\, z}{1+\chi_m}\left(1-\frac{z}{d}\right)+H_0,$$

$$p = p_b(z) = \rho_0 g d\left(\frac{z}{d}-\left(\frac{z}{d}-\frac{z^2}{2d^2}\right)\alpha\Delta T\right),\ q_{ib}=(0,0,0) \text{ and}$$

$$\rho_b(z) = -\rho_0\left(\left(\frac{z}{d} - 1\right)\alpha\Delta T - 1\right). \tag{9}$$

In the perturbed state

$$T = T_b + T', \qquad M = M_i' + M_b\ , q_i = q_{ib} + q',\ \ \rho = \rho' + \rho_b,$$

$p = p' + p_b$, and $H = H_i' + H_b$,

where the quantities with prime in the above expressions indicate the perturbed quantities.

Taking $(x, y, z) = (dx^*, dy^*, dz^*)$, $T = \Delta T T^*$, $t = \frac{d^2}{\kappa}\ t^*$,

$\omega = \frac{\kappa}{d^2}\omega^*$ and $q = \frac{\kappa}{d}q^*$, $\phi = \frac{k\Delta T\ d^2}{1+\chi_m}\phi^*$,

dimensionless equations by dropping primes and asterisks are:

$$\left(1 + \Lambda_1\frac{\partial}{\partial t}\right)R\left[M_1 J\left(\frac{\partial\phi}{\partial z}, T\right) - \frac{\partial T}{\partial x}\right] + \frac{1}{Pr}\left(1 + \Lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - RM_1\left(1 + \Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial T}{\partial x}\right)\frac{\partial T_b}{\partial z}$$

$$= \sqrt{T_a}\left(1 + \Lambda_1\frac{\partial}{\partial t}\right)(1 + \varepsilon cos(\omega t))\frac{\partial v}{\partial z} + \left(1 + \Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \tag{10}$$

$$\frac{1}{Pr}\frac{\partial v}{\partial t} + \Lambda_1\frac{\partial^2 v}{\partial t^2} = -\left(1 + \Lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}(1 + \varepsilon cos(\omega t))u + \nabla^2 v + \Lambda_2\frac{\partial}{\partial t}\nabla^2 v, \tag{11}$$

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}u = \frac{\partial T_b}{\partial x}w - \frac{\partial T}{\partial z}\ w + \nabla^2 T, \tag{12}$$

$$M_3\left(\frac{\partial^2\phi}{\partial x^2}\right) + \frac{\partial^2\phi}{\partial z^2} = \frac{\partial T}{\partial z}, \tag{13}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$.

The non-dimensional parameters are:

$\Lambda_2 = \frac{\lambda_2\kappa}{d^2}$ (Scaled strain retardation parameter),

$P_r = \frac{\mu_0}{\rho\kappa}$ (Prandtl number),

$\Lambda_1 = \frac{\lambda_1\kappa}{d^2}$ (Deborah number),

$\Lambda = \frac{\lambda_2}{\lambda_1} = \frac{\Lambda_2}{\Lambda_1}$ (Elastic ratio),

$M_1 = \frac{\mu_0 {k_l}^2 \Delta T}{\rho_0 g \alpha (1+\chi_m) d}$ (Buoyancy magnetization number),

$M_3 = \frac{(1+M_0/H_0)}{(1+\chi_m)}$(non-buoyancy magnetic Number),

$R = \frac{\alpha \rho_0 g \Delta T d^3}{\mu_0 \kappa}$(Rayleigh Number).

4.0 The Lorenz Model

Lorenz model describes the motion of a fluid under Rayleigh - Bénard situation. Amount of heat transfer is quantified by considering ferromagnetic and viscoelastic parameters employing non-linear analysis. Because of the geometry in this case, it is assumed that there are no variations in the physical quantities along the y-axis. This allows the introduction of stream functions, which contain all the information about the fluid flow in this form:

$$u = -\frac{\partial \psi}{\partial z} \text{and } w = \frac{\partial \psi}{\partial x} \tag{14}$$

That is actual fluid velocity components are obtained by taking the partial derivatives of the stream function. Using the stream function in (10) - (13),

$$\Lambda_1 \frac{\partial N_1}{\partial t} = \nabla^4 \psi - \Lambda \nabla^4 \psi - N_1 , \tag{15}$$

where N_1 *is* obtained from

$$\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2\psi = \Lambda\nabla^4\psi - (\varepsilon cos(\omega t)+1)\sqrt{T_a}\frac{\partial v}{\partial z} + RM_1 \left[-\frac{\partial^2 \phi}{\partial x \partial z} + \right.$$
$$\partial T \partial x + RM1J\, \partial \phi \partial z, T + R \partial T \partial x + N1 ,$$
(16)

$$\Lambda_1 \frac{\partial N_2}{\partial t} = \nabla^2 v - \Lambda \nabla^2 v - N_2 , \tag{17}$$

$$\frac{1}{Pr}\frac{\partial v}{\partial t} = N_2 + (\varepsilon cos(\omega t)+1)\sqrt{T_a}\frac{\partial v}{\partial z} + \Lambda \nabla^2 v, \tag{18}$$

$$M_3\left(\frac{\partial^2 \phi}{\partial x^2}\right) + \frac{\partial^2 \phi}{\partial z^2} = \frac{\partial T}{\partial z} . \tag{19}$$

Using the following BCs, equations (15) to (19) are solved:

$$D\phi = \psi = \frac{\partial N_2}{\partial z} = D^2\psi = T = \frac{\partial v}{\partial z} = N_1 = 0 \text{ at} z = 0, 1. \tag{20}$$

It is possible to obtain solutions of equations (15) to (19) that satisfy the boundary conditions (20) as an infinite Fourier series which has amplitudes that change with time. Our solution has been represented as a Fourier series, with one term for stream function, N_1, v and N_2. Similarly, temperature and velocity will retain their nonlinearity with the two terms, as given in Lorenz-Saltzman's formulation below:

$$\begin{aligned} \psi &= A(t)\, sin(kx)\, sin(\pi z), \\ N_1 &= F(t)\, sin(kx)\, sin(\pi z), \\ N_2 &= G(t)\, sin(kx)\, cos(\pi z), \\ v &= H(t)\, sin(kx)\, cos(\pi z) \\ T &= C(t)\, sin(2\pi z) + B(t)\, cos(kx)\, sin(\pi z), \\ \phi &= E(t)\, cos(2\pi z) + D(t)\, cos(\pi z)\, cos(kx), \end{aligned} \tag{21}$$

where $A(t)$, $F(t)$, $G(t)$, $H(t)$, $C(t)$ and $E(t)$ are amplitudes.

An explanation regarding the number of modes and the choice of truncated Fourier series can be found in the references [23-26, 32- 34]. By substituting equations (21) in equations (15to 19) integrating after multiplying with the correct eigenfunctions for the convection cell of size $0 \le z \le 1$ and $0 \le x \le \frac{2\pi}{k}$, following set of equations can be obtained [17]:

$$\frac{dX}{d\tau} = Pr\left[\begin{array}{c}\left(1+\frac{\pi^2 Ta}{\delta^6}\right)\left(1-\frac{M_{13}}{\pi r}Z\right)Y - \Lambda X \\ -(\varepsilon cos(\omega_1\tau)+1)\,\frac{\pi^2 Ta}{\delta^6}J - (1-\Lambda)I\end{array}\right], \tag{22}$$

$$\frac{dY}{d\tau} = rX - XZ - Y, \tag{23}$$

$$\frac{dZ}{d\tau} = \frac{4\pi^2}{\delta^2}Z + XY, \tag{24}$$

$$\frac{dJ}{d\tau} = Pr(\varepsilon cos(\omega_1\tau)+1)X - \Lambda J - (1-\Lambda)L, \tag{25}$$

$$\frac{dI}{d\tau} = \frac{1}{\Lambda_1\delta^2}(X - I), \tag{26}$$

$$\frac{dL}{d\tau} = \frac{1}{\Lambda_1\delta^2}(J - L), \tag{27}$$

Where $X = \frac{\pi k A}{\delta^2\sqrt{2}}$, $Y = \pi r B$, $J = \frac{kH}{\sqrt{2Ta}}$, $I = \frac{\pi k F}{(1-\Lambda)\delta^6\sqrt{2}}$, $L = \frac{-kG}{(1-\Lambda)\delta^2\sqrt{2Ta}}$,

$\tau = t\delta^2$, $r = \frac{R}{R_s}$, $R_s = \frac{(\delta^6+\pi^2 Ta)(\pi^2+M_3k^2)}{k^2[k^2(1+M_1)M_3+\pi^2]}$, $\delta^2 = k^2+\pi^2$, $\omega_1 = \frac{\omega}{\delta^2}$,

$M_{13} = \frac{M_1M_3\pi k^2}{k^2[k^2(1+M_1)M_3+\pi^2]}$.

Note that if $\varepsilon = 0$ and $M_3 \to 0$ or if $M_1 \to 0$ then $M_{13} \to 0$, the Lorentz system has beenrecovered exactly. With suitable initial conditions, the scaled Lorenz model (22) to (27) issolved. Here Y(0) = X(0) = Z(0) = 1 = H(0) are taken as initial conditions and this problem is solved numerically using Runge-Kutta-Fehlberg 45 (RK45) procedure.

5.0 Transfer of Heat

This paper discusses the impact of rotational modulation on the heat transfer, which can be measured by Nusselt number denoted by $Nu(\tau)$and it is defined as:

$Nu(\tau)= \frac{\text{HTC1}}{\text{HTC2}} + 1$,

where HTC1 and HTC2 represent heat transport by convection and conduction respectively.

$$Nu(\tau) = \frac{\frac{k}{2\pi}\int_0^{\frac{2\pi}{k}}\left[\left(\frac{\partial T_b}{\partial z}+\frac{\partial T}{\partial z}\right)dx\right]_{z=0}}{\frac{k}{2\pi}\int_0^{\frac{2\pi}{k}}\left[\frac{\partial T_b}{\partial z}\,dx\right]_{z=0}} + 1. \qquad (28)$$

Simplifying the equation(28), obtain the Nusselt number as:

$$Nu(\tau)=\frac{2}{r}Z(\tau) + 1. \qquad (29)$$

In the next section, we will examine the results considering mean Nusselt number $\overline{Nu(\tau)}$ vs various parameters are examined.

6.0 Results and Discussion

A Khayat-Lorenz (generalized) model is derived to examine the effect of RBC with rotation modulation in VFF. Runge-Kutta-Fehlberg45 method is employed first for solving the generalized Khayat-Lorenz model. In order to solve equations (22) -(27) the following initial conditions are taken into consideration.

Z(0) = Y (0) = X(0) = J(0) = I(0) = H(0) = 1.

Here the viscoelastic parameters, Λ and Λ_1 are representing ratio of elasticity and stress relaxation due to elasticity where as ferromagnetic parameters M_3 and M_1 are representing non buoyancy and buoyancy magnetization. In this paper, the effects of different parameters on heat transfer is discussed through average Nusselt number. Prandtl number, Pr, represents the ratio of viscous to thermal diffusion. Viscoelastic fluids have a much greater viscosity than Newtonian fluids as a result of long molecules. Therefore, a higher value is assumed for Pr than Newtonian fluids. Here convection control mechanism is modulation and its effect on transfer of heat through $Nu(\tau)$ is considered. In the limiting case with $\omega_1 = 5$, Ta=0and modulation$\varepsilon = 0$, equations (22) -(27) will give the scaled magnetic Khayat–Lorenz model of Melson et al. [34].As seen from the Fig.2, the following inequalities are obtained for four different fluids, namely Newtonian, Rivlin-Ericksen, Oldroyd fluid B and Maxwell and they are in good agreement with that of Siddheshwar et al. [23]:

$Nu(\tau)^{\text{Maxwell}} > Nu(\tau)^{\text{Oldroyd-B}} > Nu(\tau)^{\text{Newtonian}} > Nu(\tau)^{\text{Rivlin-Ericksen}}$

Plot of $\overline{Nu(\tau)}$versus $\wedge$ as seen from Fig.3shows that an increase in $\wedge$ diminishes heat transfer. According toFig.3, increase in buoyancy and non-buoyancy magnetization parameters decreases heattransfer. Graph of $\overline{Nu(\tau)}$ versus $\Lambda_1$in Fig.4. shows that and increase in $\Lambda_1$enhances heat transfer.Heat transfer is decreased with an increase in Taand it is enhanced with an increase in Pr as shown in Fig.5 and Fig.6 respectively. It is evident that rotation modulation enhances the heat transport (Table 3).

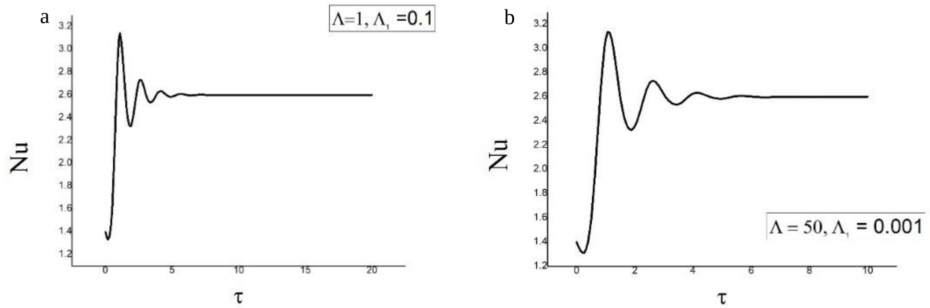
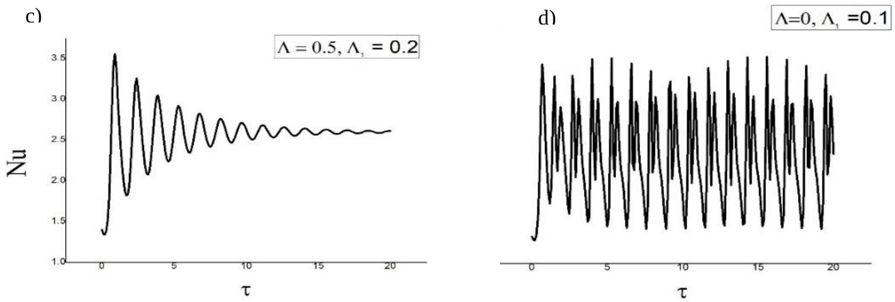


Fig.2. Plot of Nu Vs τ for P r = 10, ω_1 = 5, r = 5, Ta = 10,M_3 = 1.1 and M_1 = 10.



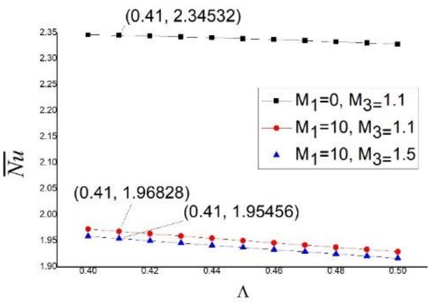


Fig. 3. $\overline{Nu}$ against Λ plot for ω_1 = 5, r = 5, Λ_1= 0.2 P r = 10, Ta = 10 , ε = 0.1.

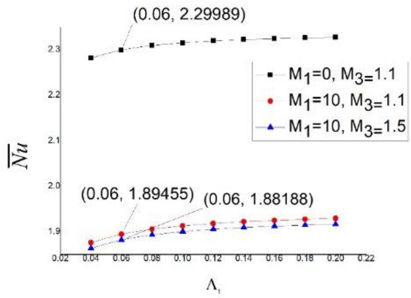


Fig. 4. $\overline{Nu}$ against Λ_1 plot with ω_1 =5, r=5, Λ = 0.5, P r = 10, Ta = 10, ε = 0.1.

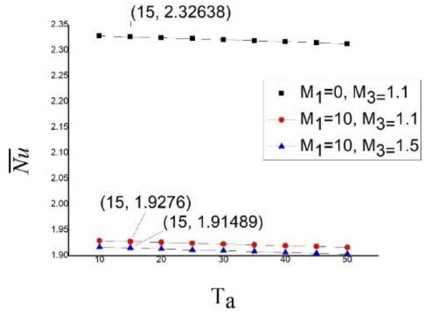


Fig. 5. $\overline{Nu}$ against Ta plot for ω_1 = 5, r = 5, Λ_1=0.2, Pr=10, Ta=10, ε=0.1, $\Lambda = 0.5$.

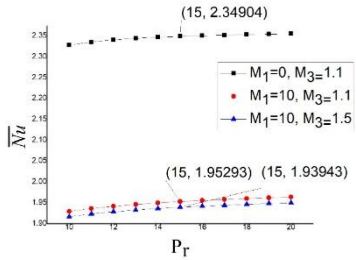


Fig. 6. $\overline{Nu}$ against Pr plot with ω_1 = 5,Λ_1 =0.2, r = 5, Pr =10, Ta = 10, ε = 0.1, $\Lambda = 0.5$.

Table 3. $\overline{Nu}(\tau)$ for $\Lambda = 0.5$, Λ_1=0.1,Pr =10, ω_1 = 5 andr = 5.

	M_3 = 1.1, M_1 = 10 and Ta=10	M_3 = 1.1, M_1 = 10 and Ta=100
	$\overline{Nu}(\tau)$	$\overline{Nu}(\tau)$
$\varepsilon = 0$	1.91222	1.893404
$\varepsilon = 0.1$	1.912905	1.898344
$\varepsilon = 0.5$	1.914236	1.909074

7.0 Conclusion

- An extended model of Khayat-Lorenz has been derived and used to study the non-linear stability effects of rotational modulation in a rotating horizontal layer of viscoelastic ferromagnetic fluids.

- Heat transfer decreases when M_1 and/or M_3 increase in the presence/absence of modulation of rotational speed.
- The stress relaxation parameter have an impact of enhancing heat transfer in the presence/absence of rotational modulation.
- Increasing strain retardation augment the heat transfer effect in the presence/absence of modulation of rotational speed.
- When the rotational modulation is present, the increase in Prandtl number increases convective heat transfer.

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